4·8·2. Indifference Curves. According to the classical theory of consumer behaviour, utility function (U) can be regarded as a function of the quantities of goods x_i, ($i = 1, 2, \ldots, n$) consumed by the consumer at any given time. Mathematically, we have

$$U = f(x, x_2, \ldots, x_n) \qquad \ldots(4{\cdot}76)$$

Let us consider the simple case in which the consumer's purchases are limited to two commodities, so that

$$U = f(x_1, x_2) \qquad \ldots(4{\cdot}76\ a)$$

where U, x_1 and x_2 are explained earlier in § 4.8. It is assumed that the function $U = f(x_1, x_2)$ is continuous and has continuous first order and second order derivatives).

Since the utility function is continuous, a given level of utility can be obtained from an infinite number of combinations of x_1 and x_2 . The locus of all combinations of the amounts of quantities consumed x_1 and x_2 for which the utility function is constant, is called an *Indifference Curve*. Mathematically, indifference curves are given by the equation :

$$U = f(x_1, x_2) = \text{Constant} = U_0, \text{ (Say)} \qquad \ldots(4{\cdot}77)$$

Note that the points on (4·76 a) represent a utility surface in 3 dimensions. The points on an indifference curve represent the various combinations of (x_1, x_2), the quantities of commodities consumed, from which the consumer gets the same utility and hence, he is indifferent to them. That is why, these curves are called indifference curves. On indifference curves, the quantity consumed of one commodity is compensated by the increase in the quantity consumed of the other commodity.

Taking total differential in (4·77), we get (on an indifference curve) :

$$dU = \frac{\partial f}{\partial x_1} \cdot dx_1 + \frac{\partial f}{\partial x_2} dx_2 = dU_0 = 0$$

$$\Rightarrow \qquad f_{x_1} dx_1 + f_{x_2} dx_2 = 0$$

$$\Rightarrow \qquad -\frac{dx_2}{dx_1} = \frac{f_{x_1}}{f_{x_2}} = \frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{MU_{x1}}{MU_{x2}} \qquad \ldots(4{\cdot}78)$$

The slope $\left(\frac{dx_2}{dx_1}\right)$ of the tangent at any point on an indifference curve is the rate at which x_1 must be substituted for x_2 (or x_2 for x_1) to maintain given level of utility to the consumer.

The negative of the slope $\left(\frac{-dx_2}{dx_1}\right)$ is the *rate of commodity substitution* or the *Marginal Rate of Substitution* (*MRS*) of x_1 for x_2 (or x_2 for x_1).

The marginal rate of substitution of commodity X_1 for commodity X_2 is defined as the number of units of X_2 that a consumer is willing to sacrifice for an additional unit of X_1 so as to maintain the same level of satisfaction.

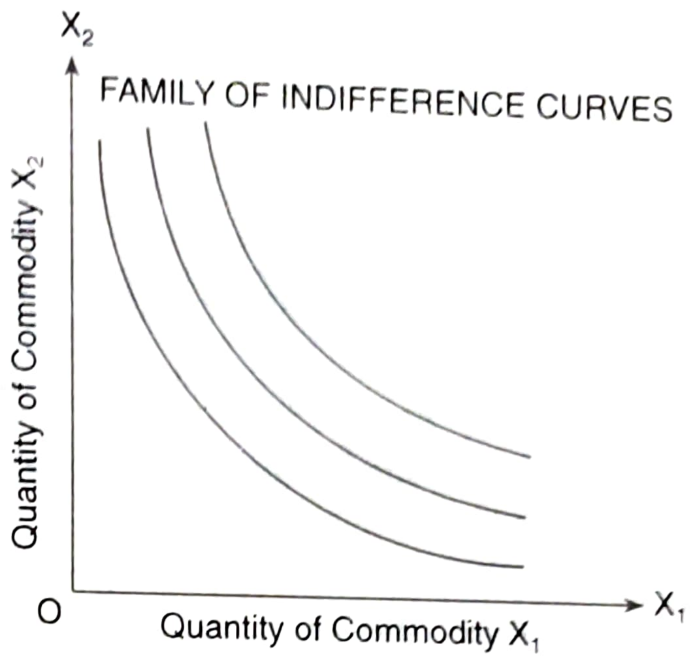

Fig. 4·10.

$$MRS\ x_1, x_2 = -\frac{dx_2}{dx_1} = \frac{MU_{x_1}}{MU_{x_2}} \qquad ...(4·79)$$

A typical family of indifference curves (which economists call an *indifference map*) is shown in *Fig. 4·10.* It may be pointed out that different consumers have different indifference curves. Moreover, the same consumer may have different indifference maps for different commodities.

Properties of Indifference Curves

(*i*) Since, $\frac{dx_2}{dx_1} < 0$, [From 4·79] the difference curve is negatively sloped.

(*ii*) Indifference curves do not intersect.

(*iii*) Indifference curves are convex from below, *i.e.*, convex to the origin.

(*iv*) An indifference curve that lies to the right of another yields more utility.

also increases.

4·8·3. Derivation of the Demand Curve using Indifference Curves. Consider the utility function $U = f(x_1, x_2)$, subject to the budget constraint $y_0 = p_1 x_1 + p_2 x_2$, the symbols having their usual meanings. The equilibrium condition for constrained utility maximisation gives :

$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{p_1}{p_2} \quad \ldots(4·80)$$

For given U, we can solve (4·80) for x_2 in terms of x_1 , p_1 and p_2. Substituting this value of x_2 in terms of x_1, p_1 and p_2. substituting this value of x_2 in the budget equation, we can finally obtain the value of x_1 in terms of p_1, p_2 and y_0, which gives the demand function for commodity X_1.

Similarly solving (4·80) for x_1 in terms of x_2 and substituting its value in the budget equation, we can find x_2 in terms of p_1, p_2 and y_0, which gives the demand function for commodity X_2.

From the equilibrium condition (4·80) we can obtain the demand functions for commodities X_1 and X_2 as explained below.

Thus , we see that the marginal utility equations (4·80) which are derived from the total utility functions, are used together with the budget equation (4·56) to get the demand functions. Hence the parameters of the utility function will determine the parametric structure of the demand functions.